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Dedicated to mathematics in general and to the following aims in particular: (1) a study of the common problems of secondary and collegiate mathematics teaching, (2) a true valuation of the disciplines of mathematics, (3) the publication of high class expository papers on mathematics, (4) the development of greater public interest in mathematics by the publication of authoritative papers treating the cultural, humanistic and historical phases of mathematics.

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CONTENTS

ARE WE SHAMMING ?

We do not put the freshman intelligence through any worthwhile educational mills when we constantly dose him with mathematical materials previously reduced to powder or tablet form by lecture process. The physical digestive machinery requires steady exercise if its powers are to be maintained at any high level, and this means that it must be furnished with the sorts of food that yield the exercise. Similarly, the power of intellectual digestion (mathematical power?) does not grow, but only shrinks under a regimen of pre-digested mental foods. Let the instructor pulverize all the lumps and knots in mathematics and the outcome can only be a flabby-minded pupil. He will be without mental initiative, without sustained concentrative ability—without either constructive or analytical power.

It is incomparably more important that the student develop ability to read a text with little or no aid from another than that he should learn to take notes from the instructor's lecture and return them to him written up in elegant but unchanged form. It is infinitely more important that he acquire the habit of doing his own thinking where humanly possible than that he should become merely adept in absorbing the thought of someone else. If nature has denied him the possession of even the beginnings of the power of independent thought, he has no place in a school—certainly is out of place in a mathematics class and should be treated as a mental defective. If he is in possession of the basis of a thinking ability—like all living things, this ability must be amenable to a proper treatment for its growth and development, and, if there is one undisputed principle in every province of nature, it is: No living thing can grow without some form of self-exercise—S. T. S.

A UNIT IN GRAPHS

By GERALDINE McCALL
Supervisor of Mathematics
Mississippi Industrial and Training School

Since the war the use and development of charts and graphs have been almost phenomenal—so extensive that we sometimes think that we as a country have gone chart mad. This development has a very solid basis in practical utility. There is little question that the chart and graph represent a genuine saving in time and mental effort. It is not alone, however, the economy of time and effort that is involved. The great thing, often, is that the chart will flash the thing not merely to the eye but to the mind. Carl Snyder says, "Charts and graphs are the most reliable and most stimulating instruments of education that we possess." Such scientific recording and projecting into the future make of business and industrial enterprise a kind of romance in reality.

The graph is seen in the advertising columns of our newspapers, in magazine articles on all sorts of economic and political questions, in various kinds of manuals used in industries, and even in the summaries of athletic records and scores of games. Graphic methods were used as an effective instrument of mathematical thinking before algebraic symbolism had developed beyond the rudiments. The Greeks never succeeded in producing a satisfactory algebraic method, yet they performed work of high importance with the aid of graphic forms.

The use of graphs has now become so common that everyone should have some idea of how to represent simple statistics by a bar or a curve-line graph. A concept of the graph is very important in elementary science, in simple mensuration, and in ordinary business. The pupil should be familiar with the types which he will meet in his reading. He must "read with" graphs even though he does not know how to construct some of the more complicated types. This means that he must develop the ability to "interpret" them.

No better subject than graphs could be chosen to illustrate the manner in which modern life demands a change in the teaching content of arithmetic. Thirty years ago the average person needed no knowledge of graphs in his ordinary reading. Now one can hardly pick up a newspaper and not find several messages and trends expressed in graphic form. Not all of these graphs are easily interpreted by one

of no experience with them. Yet they must be understood if one is to read intelligently. It therefore becomes the business of the schools to see that children are taught this new sign language expressed in lines and curves. A graph gives excellent opportunities for insisting upon the "great school virtues" of neatness and accuracy.

The young pupil should be taught the simple uses of graphs before he makes acquaintance with the more abstruse though more powerful instrument, the formula. Nunn says, "The gradual penetration of graphic methods into elementary instruction in mathematics and science may be welcomed as one of the most significant features of pedagogic tendencies."

Realizing the great importance of the subject of graphs, a unit has been entered in the junior high school course of study in mathematics at the Mississippi Industrial and Training School. What is needed in the junior high school is chiefly the reading and interpretation of both statistical and mathematical graphs, not their construction. To meet this need each class in the junior high school is required to collect graphs from any source and paste them in a large book belonging to that particular class. Under each graph is printed the name and date of publication of the paper, magazine, or journal from which the graph is taken. There is much friendly rivalry between the classes for the most attractive book and largest collection of graphs.

It is probable that the original diagrammatician lived many centuries ago, and possibly he was a cartographer. A search for him would lead us back to the days of the "Captain Kidd" legend when the word "chart" meant a sketch of an island, dead trees, and buried treasure. It would lead us back to the days of Marco Polo, whose revisions of geography upset the thinking of his contemporaries, and back to the Phoenicians who doubtless kept strange maps to guide them about the Mediterranean shores. The Egyptians 4000 years ago made floor-plans of the pyramids; and the Chinese, 6000 years ago, made maps of the heavens. Because of the antiquity of maps in the world's history the pupils began their graph books with maps and diagrams.

From the portrayal of space relation between objects by the use of maps and diagrams, we turn naturally to the portrayal of abstract ideas having no space existence. Instead of location on an actual surface, we wish to show position in an imaginary way. Consequently in the second division of the graph books we deal with a logical analysis—the classification chart.

The third division of the graph books is made up of horizontal, vertical, and pictorial bar-charts. Bar-charts are most flexible and can be varied to suit the individual whims of the maker. The most satisfactory style, or form, consists of a horizontal grouping of bars alongside the data. There should be normally but one column of bars, because the bars can be advantageously compared only when they are side by side, one below the other. Rows of figures, all drawn to the same scale, can be used in the place of bars. The proper way to show pictorially the comparison between two or more items is not to draw a small object and an enlarged replica of this beside it, but to draw one object and by it several of the same size. There are countless pictorial devices—the simplest of which is to indicate a third dimension to the bars, setting them on end for this purpose. Between the sensational picture-bar, and the plain-bar itself, there is the popular "pipe-organ chart," or the vertical-bar chart. The chief value of this chart lies in the realistic picture it gives of quantities. We do not have to climb up and get a bird's eye view of them as in the ordinary bar-chart, where we seem to be looking down upon rows and rows of goods, but we see them from a natural point of view.

The circle charts, or pie diagrams, make up the fourth division of the graph books, and there is no chart which is more purely popular in appeal. For analytical purpose, it has nothing to recommend it, but for sensational values it is, in general, without an equal. Centuries ago it was a mooted question among philosophers whether the Lord could make a yardstick which was endless. Then some one suggested that the yardstick be bent into circular form and the question was dropped. The circle graph is worthless for study and research purposes because the various parts cannot be easily compared. But a circle graph instantly commands the reader's attention, and this fact is well known to advertisers.

In the fifth division of the graph books are curves. The person who invented curves was not necessarily clever, but was assuredly lazy. Since it was only the ends of the bars he was interested in, he said, "I will place a dot where each bar ends, connect these dots with lines, and let it go at that."

In the sixth and seventh divisions of these graph books are the cyclograph and the third dimension graph.

The abilities that are required of all junior high school pupils with respect to this topic are:

(1) Make simple statistical tables.

(2) Make and read such tables as those showing correlation between height, weight, and age.

(3) Draw simple bar, line, and circle graphs, (The pupil is shown here that a graphic record is useless unless its object is clearly stated and unless the scales of representation are properly labelled and graduated.)

(4) Interpret graphs of the various kinds in the graph books mentioned above.

(5) Criticize graphs that give a false impression of the statistics.

(6) Decide upon the best type of graph for a given set of statistics.

(7) Locate points with respect to two perpendicular axes, OX and OY, using X for the abscissa and Y for the ordinate. (The pupils are told here that the system of coordinates originated with Descartes in 1637. His system is the one known to us as Cartesian coordinates. The nearest approach before Descartes to the use of coordinates as we know them was made by Appollonius about two centuries before Christ.)

(8) Understand and use directed numbers in connection with graphs.

Graphs can be taught from the chapter given on them in the text book but the teacher who uses the text book exclusively for a source book and for a motivating device will fail to accomplish the desired results. The board of education might save such a teacher's salary by assigning some one to give the pupils an examination on the book when they have completed the study of it at home. On the other hand if the work on graphs is motivated in the manner already discussed in this paper, the subject is so enlivened that the class receives an inspiration. This being true, when the pupils come to algebra, the study of it is lifted above the dry pages of the text book and the pupils are able to see very readily the practical value of such a study.

Bibliography

Hassler and Smith—*The Teaching of Secondary Mathematics,* The Macmillan Company, 1930.

Karsten—*Graphs.*

Nunn—*The Teaching of Algebra,* Longmans, Green and Co., 1927.

Sanford—*A Short History of Mathematics,* Houghton Mifflin Co., 1930.

Smith and Reeve—*The Teaching of Junior High School Mathematics,* Ginn and Co., 1927.

THE TIMING OF HIGH SCHOOL ALGEBRA AS RELATED TO SUCCESS IN COLLEGE ALGEBRA

By J. BRUCE COLEMAN
University of South Carolina

A canvass of the Freshman class at the University of South Carolina shows the following results for 1931:

Group I contains 183 students who have taken either a regular, or a review, course in algebra during the last year of high school.

Group II contains 136 students who did not take any algebra during the last year of high school. (Most of this group had algebra in the 8th and 9th grades.)

Percentage of group with indicated mid-semester grade in Freshman Algebra.

Group	*A-B-C*	*D*	*E*
I	59	24	17
II	34	31	35

(E indicates a total failure. D might be considered an unsatisfactory pass. A, B and C show generally satisfactory work.)

The canvass of 1930 showed 19% of group I as having a grade of E, while 48% of group II had a grade of E.

* * * * * * * * * * *

A multiplicity of causes is naturally involved in the marked difference between the college records of the two groups of students. One of these is, undoubtedly, the fact that the students of group I took algebra when they were more mature than was the case generally with those of group II. Another is the fact that the students of group I did not have a long period of forgetting between the study of high school algebra and that of college algebra. Whatever may be your interpretation of these results, I believe you will find the differences significant as regards preparation for College Algebra.

Natchitoches, Louisiana

February 10, 1932.

Dear Professor Sanders:

I have the following for the Section program for Friday p. m., March 11:

An Elementary Problem in Vector Analysis,
Professor T. J. Lindsey, Mississippi State College.

A Study of Certain Problems from the Field of Investment,
Professor I. C. Nichols, Louisiana State University.

Small Oscillations of the Neutral Helium Atom Near the Straight Line Positions,
Professor H. E. Buchanan, Tulane University.

On Cayley's Formulas for Orthogonal Determinants,
Professor Dewey S. Dearman, Mississippi State Teachers College.

Professor Jackson will discuss "Trigonometric Interpolation" some time during the Section program Friday afternoon. His main address will be given Friday evening on "Least Squares and Shortest Distances."

Sincerely yours,
A. C. MADDOX.

Department of Mathematics
University, Mississippi
Box 422

February 12, 1932.

Dear Professor Sanders:

At the suggestion of Professor A. C. Maddox I am writing you relative to hotel accommodations here during our meeting on March 11-12. There is only one hotel in Oxford, the Colonial. The manager of this hotel has kindly agreed to have a representative in the lobby of the Graduate building on March 11th, to make assignment of rooms to those desiring hotel accommodations. He can comfortably sleep fifty people, so he informs me, but not more than this number. Should there be more than fifty desiring hotel accommmodations the over-plus will be assigned rooms in private homes. It is for this reason that headquarters are being set up in the lobby of the Graduate building rather than at the hotel.

Those attending the meeting are to report at the desk in the lobby of the Graduate building for room assignments. Those expecting to attend should write me so signifying their intention in order that necessary arrangements may be made to insure their comfort, I will greatly appreciate same. And, finally, if those who expect to attend the banquet on the evening of the 11th (plates, $1.00) will be so kind as to notify me in advance necessary arrangements can be made accordingly.

With best wishes and a cordial welcome to "Ole Miss".

M. C. RHODES.

APPARENTLY A SUCCESSFUL PRESENTATION

By M. W. COULTRAP
North Central College
Naperville, Ill.

Some years ago when I was Superintendent of a city school in Indiana, a sixth grade teacher accosted me one noon-recess in the following words: "I wish you would come into my room and see what is the matter with my pupils; I have been trying for *three weeks* to teach them how to find the area of a floor, and not one of them can do it."

I did not give expression to the thought that passed through my mind, which was, "What is the matter with the teacher?"

I requested her to pick out the three whom she regarded the poorest and send them to my office at 4 o'clock.

This she did. In making preparation for my work, I secured a large rectangular board 30 by 42 inches. I realized that I must teach the subject without their knowing that I was doing so, for if I mentioned the subject, they would immediately drift into the old way of thinking, which had gotten them nowhere. Hence I planned my work so that they would get the idea they were helping me with some work I was interested in, and needed to get done at once. I had placed the rectangular card board on the table, and asked them to come and help me with my work for a little while. They came very quickly, and felt glad to be asked to help.

As soon as they came around the small table I said, "I want you to help me to find out some way of finding the number of squares on this large card board without counting all of them."

After they had examined the board a short time I said, "I wonder how many squares in that first row running lengthwise." They counted through and said 42. Then I repeated several times aloud as though I were thinking, "42 squares in the first row." Having done this till I thought they had it clearly in their minds, I said, "I wonder how many squares in the second row?" I really expected them to say 42, but they did not; they counted clear through and then said 42.

Then I repeated aloud several times, as though I were talking to myself, "42 squares in the first row, and 42 squares in the second row."

Having repeated this some four or five times, I said, "I wonder how many squares in the third row." Again to my surprise they com-

menced counting. After counting about half way through, one stopped and suddenly said, "There would be 42, would there not?"

I said, "Would there?" "Yes," he said, "there would be as many in one row as another." Immediately the other two agreed. I remarked, "I believe that would be so." Then all three in one voice said, "Sure, there would be as many in one as another." I then admitted that I was quite sure they were correct. However, to make sure that they had this idea firmly fixed in their minds, I said, "How many in the fourth row?" In one voice, they all said, with something of *disgust* in their voices, "42. As many in one row as another, 42 in each row."

I was now thoroughly convinced that this important idea was well fixed in their minds. Then I said, "How many rows are there?" They counted, and said "30."

Then I said, "How many squares would there be on the card board?"

After thinking a moment, they said, "30 times 42 squares, 1260 squares."

So far I had not said a word about the area of a floor. Next, I said I would like to play a little game with them.

This would be a game in which they would have to shut their eyes and keep them shut through the entire game. They were anxious to play the game, and agreed not to open their eyes.

I then said, "You did not observe when you came into this room"—here I had to caution them several times that I could not play the game properly if they did not keep their eyes closed tightly during the entire game. They having assured me that they would not open their eyes, I started again.

"You did not observe red lines drawn across this room east and west 1 foot apart, and blue lines drawn north and south 1 foot apart" (of course there were no lines on the floor, hence I wanted their eyes closed so they would have to see the lines mentally.)

"What does the floor look like?" With their eyes still closed, they said "A Checker board."

I then told them that the room was just thirty feet long and twenty feet wide, which was true.

Then I asked, "How many squares in the first row lengthwise?" They answered promptly, "30". "How many in the second?" The answer was "30, as many in one row as another." "How many rows"

"20". "How many squares on the floor?" Promptly they said, "20 times 30 squares, 600 squares." So far I had not said *area.*

Then I asked them, "If they didn't now think they could find the *area* of a floor."

Almost with a gasp, they said, "Is that all there is to it?"

I responded, "That is all I know about it," and permitted them to go home.

The next morning at recess their teacher told me that those three pupils could solve problems in finding areas of floors as fast as she gave them, and they were the only ones in the class of seventeen who could.

Their future work showed that they could find areas of floors *quickly* and *accurately.*

Doubtless you are wondering what was the method their teacher had been trying for three weeks with no good results.

I found out later, and will give it if the readers of the NEWS LETTER desire me to do so.

AXIOMS AND THEIR RELATIONS TO SECONDARY SCHOOL MATHEMATICS*

By DOROTHY McCOY
Belhaven College

The propositions of mathematics are proved as logical deductions from previously proved propositions and they in turn are proved from other propositions. Obviously there must be a beginning with some unproved assumptions. These assumptions are called axioms.

This statement of what axioms are indicates the usual order of development of mathematical theory. That is, only after a considerable body of material is at hand does it seem worth while to discover just what assumptions were made. For a historical example of such a development Euclid's chief contribution to plane geometry was systematic organization from his axioms and postulates as most of the propositions he used were previously known. Also the definition would suggest that with a large number of assumptions theorems usually coming later in the sequence might be proved immediately, which of course is true.

*Presented to the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics, March, 1931.

When there arise questions concerning what to use as assumptions, whether or not all the assumptions have been stated, or whether there are any duplications or contradictions among our assumptions we are beginning a very interesting study of axioms themselves. Is our list of axioms consistent? If so there are no contradictions among them and we can prove no contradictions by their use. Are they independent? Yes, if no one of them can be obtained as a logical consequence of one or more of the other axioms. Proof of independence is easier talked about than done. Think of one of the axioms of geometry and try to prove it from the others. Perhaps you see no way of deducing it but then try to prove that it cannot be done! Mathematicians give such proofs using what is known as existential examples and finding them is like making up a parable except the example must fit the case exactly whereas parables usually do not fit in all their details. But we cannot say we have finished our study of axioms even here. To study *complete independence* existential examples to the number of 256 are necessary if we have eight axioms and double that number of examples are necessary should we be so inconsiderate of our time as to begin a study of nine axioms.

Frequently we find that more assumptions are used in a given theory than are stated as such. This is not only true of Euclid and other early writers but additional cases come to light from time to time. For example a widely used college algebra gives the eleven axioms which I will state symbolically.

1. $a+b=x$
2. $a+b=b+a$
3. $(a+b)+c=a+(b+c)$
4. $a=b,\ c=d,\ a+c=b+d$
5. $ab=y$
6. $ab=ba$
7. $(ab)c=a(bc)$
8. $a(b+c)=ab+ac$
9. $a=b,\ c=d,\ ac=bd$
10. $x+b=a$
11. $bx=a\ (b\neq 0)$

Anyone interested in doing so can verify that these axioms do not require the algebra to have more than two numbers by taking two numbers having the properties of zero and one, except that for these numbers the sum of $1+1$ is to be zero. When an algebra of more than

two numbers is used with these axioms obviously some additional assumption is made. This is also an illustration of an existential example.

Another interesting fact about axioms is that they do not have to appear self evident or even reasonable. For instance in a baseball game the score may not be the same if player A bats before player B as it would be with player B batting first, whereas in our algebra we say that x plus y is the same as y plus x and that it is self evident. Similarly we know that this axiom does not hold for movements of rigid bodies. Euclid included the axiom known as the parallel postulate and since that time non-Euclidean geometries have been developed omitting that axiom or making other assumptions than that one. Denying the parallel postulate may not appear reasonable yet valuable theory is deduced without it. The recent theory of quantum mechanics as developed by Dirac is based upon assumptions which do not appear reasonable, nevertheless by its use physicists are able to predict accurately the results of experiments. In this theory of Dirac multiplication is not commutative that is $ab \neq ba$ while in all our elementary mathematics we use either factor as multiplier. These illustrations show also that axioms do not necessarily hold in every phase of life.

Have you seen axioms for distance, space, equality, order, betweenness, dimension or for natural numbers? Just what assumptions or axioms could be involved when we speak of the distance between two points? Here are three axioms often used. First, the distance between two coincident points is zero; second, the distance between two non-coincident points is greater than zero; and, third the sum of the distances from x to y, and y to z is greater than the distance from x to z. These are obvious and interesting properties of distance as we know it. It is surprising how much theory can be developed from these axioms. Each of the other concepts mentioned, space, order, etc., are developed from equally interesting axioms.

What bearing do these facts have upon teaching in the secondary school?

In the first place the historical development of axioms after a considerable body of material was known would suggest that pupils should have some knowledge of the field before its development from axioms is stressed. Likewise, as a large group of axioms is usually found first and only after much study reduced to a minimum number the pupil should not be required to use the fewest possible axioms.

He can learn the methods of mathematics equally well using a large number of axioms which is psychologically to be preferred. When only a few axioms are used it becomes necessary for pupils to prove propositions which appear to him quite obvious and which should therefore be included in his assumptions.

Algebra has axioms as well as plane geometry and it seems to me that this field is much neglected. A pupil who learns to "cross multiply" loses many good opportunities to see that an equality results when equals are multiplied by equals. Test your advanced classes and see how many know why cross multiplying works! Perhaps many pupils would be better prepared for geometry were they shown that an axiom is applied when they multiply a times b instead of b times a. Many similar opportunities for pointing out axioms repeatedly occur in algebra.

Even though there are opportunities for teaching the place of axioms in algebra, plane geometry is so organized as to be much easier for this. It is a thrilling experience to see how the body of theorems of plane geometry develops from so few assumptions. It is to be hoped that every pupil may appreciate this experience to the fullest.

RATE OF INTEREST PAID ON A CERTAIN INSTALLMENT PURCHASE

By IRBY C. NICHOLS
Louisiana State University

[*It is hoped that this article will be read by all Senior High School students in particular.*]

When money is easy, the credit of the buying public is good and profits in business are satisfactory—neither buyer nor seller complains. But in times of economic depression, every phase of business undergoes a re-examination and both buyer and seller become critical: The margin of profit is cut and credit buying, in particular, is given special attention—more than ever before installment buyers ask about rates of interest paid on their balances.

To this end, a certain buyer writes, asking "what effective rate of interest am I paying on a balance due on a radio of $144.00, if I give the seller my note for $152.64 ($144.00 plus 6% of itself) to be paid in

twelve installments of \$12.72 each, the first installment to be paid one month after date, the second two months after date, and so on until all twelve installments are paid?"

This question is typical of the times. Here the buyer knows that he is paying more than 6% per annum as interest on his balance due, and frankly asks the *exact* rate being paid.

Solution. *First Method.* If a clerk in a financial institution be asked to answer this question, he will most probably use the traditional ***bank discount*** method, and take as the time 6.5 years applied to one payment. He will argue that since the seller waits one month for a payment of \$12.72, two months for a payment of \$12.72, three months for a like sum, and so on, he waits, in the aggregate, the equivalent of 78 months, or 6.5 years for a payment of \$12.72. The total *discount* paid is 6% of \$144.00, or \$8.64. Hence $\$12.72 \times i \times 6.5 = 8.64$, where i is the rate of interest sought; whence $i = 10.4\%$ per annum, as obtained by ***bank discount***.

Second Method. In solving this same problem, the public school teacher would very probably use *true* discount,and, incidentally, use the *average* time of one payment and apply it to the sum of all the payments: The first payment is to be made one month after date, the last payment twelve months after date, and all of them at regular intervals, forming an arithmetic series. The arithmetic average of the time is 6.5 months, or $^{13}/_{24}$ years. The balance due is \$144.00; the total amount to be paid is \$152.64, payable monthly over a period of twelve months, but averaging 13/24 years for the whole. Therefore $144 = 152.64/(1 + {}^{13}/_{24}i)$, from which the purchaser readily obtains $i = 11.07\%$ as the rate of interest figured by the *true discount* method, using *average* time.

In the first method, notice that the *total time*, 6.5 years, of all the payments is applied to one *average monthly payment*;but in the second method, the *average time*, $^{13}/_{24}$ years, for one payment is applied to the *total of all the payments*. Notice that the *average* here used is the *arithmetic* average. Obviously both methods are in error: A *monthly turn-over* of money is involved in the problem, which is a *geometric* property and not an *arithmetic* one. The second method gives a rate of interest not far from the correct rate, but the first method contains an error which widens considerably for long periods of time; for instance,consider by this method the proceeds of a note, discounted for $12\frac{1}{2}$ years at 8% interest. All of the note would be consumed in the discount,an absurd idea. The method is used very generally

because it furnishes a rule simple to apply and easy to explain. The usual clerk can handle it. However, the question still remains as to the correct rate of interest paid in the purchase cited above. Let us turn to the student of investments.

Third Method. When the student of the mathematical theory of investment is asked for an answer to questions of this type, the standard formula $A=P(1+i)^n$, or $P=A/(1+i)^n$, presents itself to his mind. P is the present value of an amount A due in n years at rate of interest i. Using this formula, the ***present value of each monthly payment*** is obtained ***separately***, since each payment has its own respective time interval; the sum of several present values of the separate payments equals the present value of the whole. Hence the equation

$$144=12.72/(1+i)^{1/12}+12.72/(1+i)^{2/12}+12.72/(1+i)^{3/12}$$
$$\ldots\ldots +12.72/(1+i)^{12/12}.$$

Factoring out 24 and using negative exponents, this becomes

$$6=.53[(1+i)^{-1/12}+(1+i)^{-2/12}+(1+i)^{-3/12}+\ \ldots\ldots\ +(1+i)^{-12/12}].$$

The right hand member of this equation is a geometric series whose first term a is $(1+i)^{-1/12}$, whose ratio r is $(1+i)^{-1/12}$ and whose last term is $(1+i)^{-12/12}$. The formula for the sum of a geometric series is $(rl-a)/(r-1)$. Applying it here, one obtains

$$6=.53\left\{\frac{(1+i)^{-13/12}-(1+i)^{-1/12}}{1+i)^{-1/12}-1}\right\}$$

Whence $6(1+i)^{-13/12}-6.5i-6=0$. Expanding the first term of this last equation by the binomial theorem and dropping all terms containing i with a power greater than three, since such terms are a decimal too small to affect the result sought within four decimal places, one now has

$$143i^3-468i^2+51.84i=0.$$

Factoring, $i=0$ and $143i^2-468i+51.84=0$. The second factor gives $i=3.14$ and $.114$ manifestly $i=0$ and $i=3.14$ do not satisfy the original conditions of the problem under discussion. Therefore $i=.114$ or 11.4%. This then is the the effective rate of interest paid by the purchaser of the radio on his balance due of \$144.00 if paid in installments as indicated above.

FINESSE AT BRIDGE

By S. T. SANDERS, Jr.
St. Joseph, Mo.

The finesse at bridge should appeal to one interested in elementary probability theory.

The play often occurs as follows: The declarer, holding the ace and king of spades, leads the jack from dummy. If the second player holds the queen and plays, the king takes the trick. If the queen is not played, the king is withheld, and the jack remains high, provided the queen does not lie in the fourth hand. Such a finesse evidently has one chance out of two of succeeding, since the two opposing hands have equal chances of holding the missing queen.

The questions arise: When should one finesse rather than attempt to down the queen by leading successively the ace and king? And what is the probability of the success of this latter method in a particular case?

Let us assume the declarer holds 8 spades, including the ace, king, and jack. Then there are 5 held against him. Now, 5 cards may be distributed in 2^5 ways—a result yielded by the binomial expansion when the value unity is assigned to each of the two constants.

Of these 2^5 ways in which the adverse spades may be located, let us determine those distributions which permit the capture of the missing queen in two rounds (by ace and king).

If the five are divided 5 and 0, the queen is safe. However, a $4-1$ division insures her safety only if she is one of the four. We have then, thus far, $1=C(4,0)$ ways in which she may fall.

If the spades lie three in one hand—two in the other, the queen escapes if she is of the three—is trapped if she lies in the doubleton. In the latter event, she may choose her companion from four; hence the ways of her downfall are $C(4,1)=4$.

We have, in all, $C(4,0)+C(4,1)=5$ ways of downing the queen, but this number must be doubled, since the cards may fall $2-3$, $1-4$, or $0-5$. Our chance, then, by this method of play is $\frac{10}{2^5}=\frac{5}{16}$, whereas the finesse gives a probability of success $=\frac{1}{2}$.

Should there be an even number of spades, say 4, held against the declarer, we may reason as before for the distributions $4-0$, $0-4$, $3-1$,

and $1-3$. As for the $2-2$ arrangement, the queen in either hand will drop to the ace, king. Now, $C(4,2)=6$ is obviously the number of $2-2$ possibilities, but for the sake of uniformity, let us replace $C(4,2)$ by its equal, $2C(3,1)$. Our total chance then becomes

$$\frac{2[C(3,0)+C(3,1)]}{2^4}=\tfrac{1}{2}.$$

As for the general case, setting

$A-n=$card missing, ($A=$ace)

and $2n+k=$number of cards adversely held, there evidently remains only the adaptation of our method to the distribution, $\dfrac{2n+k}{2}$, $\dfrac{2n+k}{2}$, where k, of course, is even. It is easily shown that

$$C(2n+k, \frac{2n+k}{2})=2c(2n+k-1, \frac{2n+k}{2}-1).$$

We have, therefore, the following probability for the total of ways of dropping the missing card:

$$2[C(2n+k-1,0)+C(2n+k-1,1)+ \; . \; . \; . \; . \; +C(2n+k-1,n-1)]$$

Then, since the number of ways of distribution is 2^{2n+k}, we finesse when and only when our probability of successfully leading out,

$$\frac{C(2n+k-1,0)+C(2n+k-1,1)+\ldots+C(2n+k-1,n-1)}{2^{2n-k-1}}\leqq\tfrac{1}{2}.$$

That is, when $\sum_{n=0}^{n-1}C(2n+k-1,r)\leqq 2^{2n-k-2}$.

But this is true when and only when $K\geqq 0$, since, taking $k=0$, we have by the same application of the binomial theorem:

$$\sum_{n=0}^{2n-1}C(2n-1,r)=2^{2n-1};$$

But since $2n-1$ is odd,

$$\sum_{n=0}^{n-1} C(2n-1,r) = \tfrac{1}{2}\sum_{n=0}^{2n-1} C(2n-1,r) = 2^{2n-2};$$

Again, for $k>0$ we have, similarly,

$$\sum_{n=0}^{2n+k-1} C(2n+k-1,r) = 2^{2n-k-1};$$

$$\text{But}\quad \sum_{n=0}^{n-1} C(2n+k-1,r) < \tfrac{1}{2}\sum_{n=o}^{2n+k-1} C(2n+k-1,r) = 2^{2n+k-2}.$$

Accordingly, we finesse when and only when the number of adverse cards is greater than or equal to double the number of top cards in our combined hands.

SOME APPLICATIONS OF DETERMINANTS TO GEOMETRY*

By ROBERTA IZARD and MARY ELIZABETH WILSON
Mississippi Woman's College

There are many applications of determinants to geometry and some of these are found in almost every book on analytic geometry. There are, however, some rather simple applications which are not usually given and which we believe might be of interest to those of our readers who may not be familiar with them. In this paper we shall consider the formula for the area of a triangle when we are given, (1) the coordinates of its vertices, (2) the equations of its sides, (3) the lengths of its sides. Most of material here may be found in Kowalewski, Determinantentheorie, chapter 15.

Let (x,y) be the coordinates of a general point referred to a rectangular system. If 2 points (x_1,y_1) and (x_2,y_2) are given, the equation of the line through these points may be written as:

$$(1)\qquad \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

*The authors appreciate the aid given by Dr. W. V. Parker, head of the Mathematics Department of Mississippi Woman's College.

The distance from the line (1) to a third point (x_0,y_0) is $|h|$, where

$$h=\frac{\begin{vmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}$$

The distance between the points (x_1,y_1) and (x_2,y_2) is

$$g=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}.$$

The area, J, of the triangle, the coordinates of whose vertices are (x_0,y_0), (x_1,y_1) and (x_2,y_2) is, therefore, $\frac{1}{2}|gh|$, that is J is equal to the absolute value of

$$(2) \qquad \frac{1}{2}\begin{vmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}$$

While this formula is ordinarily found in text books, the analogous formula for three sides given is not usually given and may be obtained as follows:

Suppose the equations of the three sides are:

$$(3) \qquad \begin{array}{ll} (l_0) & a_0x+b_0y+c_0=0, \\ (l_1) & a_1x+b_1y+c_1=0, \\ (l_2) & a_2x+b_2y+c_2=0. \end{array}$$

Denote by (x_i,y_i) the vertex opposite $l_i (i=0,1,2)$. If then we denote by A_i,B_i,C_i, the cofactors of a_i,b_i,c_i in the determinant

$$\Delta=\begin{vmatrix} a_0 & b_0 & c_0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix},$$

From (3), we get $x_i=\dfrac{A_i}{C_i}$, $y_i==\dfrac{B_i}{C_i}$.

The area of the triangle having these lines as sides is, therefore, by (2) equal to the absolute value of

$$\frac{1}{2}\begin{vmatrix} \dfrac{A_0}{C_0} & \dfrac{B_0}{C_0} & 1 \\ \dfrac{A_1}{C_1} & \dfrac{B_1}{C_1} & 1 \\ \dfrac{A_2}{C_2} & \dfrac{B_2}{C_2} & 1 \end{vmatrix} = \frac{1}{2c_0c_1c_2}\begin{vmatrix} A_0 & B_0 & C_0 \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$$

$$= \frac{1}{2}\frac{\begin{vmatrix} a_0 & b_0 & c_0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}^2}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \cdot \begin{vmatrix} a_2 & b_2 \\ a_0 & b_0 \end{vmatrix} \cdot \begin{vmatrix} a_0 & b_0 \\ a_1 & b_1 \end{vmatrix}}.$$

(See Bocher, Introduction to Higher Algebra, Corollary 2, page 33.)

From (2) we have

$$J^2 = \frac{1}{4}\begin{vmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}^2,$$

Which may be written

$$J^2 = \frac{1}{4}\begin{vmatrix} x_0 & y_0 & 1 & 0 \\ x_1 & y_1 & 1 & 0 \\ x_2 & y_2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}^2$$

$$J^2 = -\frac{1}{4}\begin{vmatrix} x_0 & y_0 & 1 & 0 \\ x_1 & y_1 & 1 & 0 \\ x_2 & y_2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}\begin{vmatrix} x_0 & x_1 & x_2 & 0 \\ y_0 & y_1 & y_2 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}.$$

Multiplying row by column, we get

$$(4) \qquad J^2 = -\frac{1}{4}\begin{vmatrix} x^2_0+y_0^2 & x_0x_1+y_0y_1 & x_0x_2+y_0y_2 & 1 \\ x_0x_1+y_0y_1 & x_1^2+y_1^2 & x_1x_2+y_1y_2 & 1 \\ x_0x_2+y_0y_2 & x_1x_2+y_1y_2 & x_2^2+y_2^2 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}.$$

If d_{ij} denotes the distance between the points (x_i,y_i) and (x_j,h_j) we have

(5) $d^2_{ij}=(x_i-x_j)^2+(y_i-y_j)^2$, $(i,j=0,\ 1,\ 2)$.

From (5) we get

$$x_ix_j+y_iy_j=-\tfrac{1}{2}\,d^2_{ij}+\tfrac{1}{2}(x^2_i+y^2_i)+\tfrac{1}{2}(x^2_j+y^2_j).$$

If in the determinant (4) we subtract from the$(i+1)$st row$(i=0,1,2)$ the last, multiplied by $\frac{1}{2}(x^2_i+y^2_i)$, and from the $(j+1)$st column $(j=0,1,2)$ the last, multiplied by $\frac{1}{2}(x^2_j+y^2_j)$ we get

$$J^2=-\tfrac{1}{4}\begin{vmatrix} 0 & -d^2_{01} & -\frac{1}{2}d^2_{02} & 1 \\ \frac{1}{2}-d^2_{01} & 0 & -\frac{1}{2}d^2_{12} & 1 \\ -\frac{1}{2}d^2_{02} & -\frac{1}{2}d^2_{12} & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

If now we multiply the first three rows by -2 and then the last column by $-\frac{1}{2}$ we get

$$J^2=-\tfrac{1}{16}\begin{vmatrix} 0 & d^2_{01} & d^2_{02} & 1 \\ d^2_{01} & 0 & d^2_{12} & 1 \\ d^2_{02} & d^2_{12} & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

If now we write $d_{12}=a_0,$
$d_{20}=a_1,$
$d_{01}=a_2,$

We have

$$J^2=-\tfrac{1}{16}\begin{vmatrix} 0 & a_2^2 & a_1^2 & 1 \\ a_2^2 & 0 & a_0^2 & 1 \\ a_1^2 & a_0^2 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

$$=-\tfrac{1}{16}\begin{vmatrix} 0 & a_2^2 & a_1^2 & 1 \\ a_2^2 & -a_2^2 & a_0^2-a_2^2 & 1 \\ a_1^2 & a_0^2-a_1^2 & -a_1^2 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

$$=-\tfrac{1}{16}\begin{vmatrix} 0 & a_2^2 & a_1^2 & 1 \\ a_2^2 & -2a_2^2 & a_0^2-a_1^2-a_2^2 & 0 \\ a_1^2 & a_1^2-a_1^2-a_2^2 & -2a_1^2 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

$$=\frac{1}{16}\begin{vmatrix} -2a_2^2 & a_0^2-a_1^2-a_2^2 \\ a_0^2-a_1^2-a_2^2 & -2a_1^2 \end{vmatrix}$$

$$=\frac{1}{16}[4a_1^2a_2^2-(a_0^2-a_1^2-a_2^2)^2]$$

$$=\frac{1}{16}[2a_1a_2-(a_0^2-a_1^2-a_2^2)][2a_1a_2+(a_0^2-a_1^2-a_2^2)]$$

$$=\frac{1}{16}[(a_1^2+2a_1a_2+a_2^2)-a_0^2][a_0^2-(a_1^2-2a_1a_2+a_2^2)]$$

$$=\frac{1}{16}(a_1+a_2-a_0)(a_1+a_2+a_0)(a_0-a_1+a_2)(a_0+a_1-a_2).$$

If we write

$$S=\frac{a_0+a_1+a_2}{2},$$

We have the well known formula for the square of the area of a triangle in terms of the lengths of its sides, namely

$$J^2=s(s-a_0)(s-a_1)(s-a_2).$$

NOTE ON THE VOLUME OF A TETRAHEDRON

By H. L. SMITH
Louisiana State University

The volume of a region in Riemann geometry is usually defined by means of a certain integral, the definition being justified by the invariance of the integral under change of coordinate system and by the fact that it reduces in the case of Euclidean geometry to the familiar one. In the course of attempting to introduce the notion of volume into Riemann geometry on a purely geometric basis, the writer was led to consider the formula for volume of a tetrahedron in terms of the lengths of its edges. The reader will find such a formula in Kowalewski's well known treatise.* It involves a determinant of the fourth order. In this note, the writer derives a formula in terms of a determinant of the third order. It is very easily obtained on the basis of the results of the writer's recent note on vectors in this NEWS LETTER, vol. 6, No. 2, pp. 18-23. We proceed to the derivation.

*Die Determinantentheorie, p. 348.

Let P_0, P_1, P_2, P_3, be the vertices of a tetrahedron and let a_{rs} be the length of edge $P_r P_s$. Let V_1, V_2, V_3, be the vectors $P_0 P_1$, P_0P_2, P_0P_3, respectively. Then

$$\text{Vol. } P_0P_1P_2P_3 = (|V_1, V_2, V_3|^2)^{\frac{1}{2}}$$

Hence, by section 8 of the paper just referred to,

$$(1) \quad \text{Vol. } P_0P_1P_2P_3 = \begin{vmatrix} V_1V_1 & V_1V_2 & V_1V_3 \\ V_2V_1 & V_2V_3 & V_3V_3 \\ V_3V_1 & V_3V_2 & V_3V_3 \end{vmatrix}^{\frac{1}{2}}$$

But by definition of scalor vector multiplication,

$$V_rV_s = a_{or}\, a_{os} \cos \angle P_rP_oP_s.$$

Hence if s = r, we have

$$(2) \quad V_rV_r = a_{or}^2$$

If $s \neq r$ we have by the law of cosines

$$a_{rs}^2 = a_{or}^2 + a_{os}^2 - 2a_{or}a_{os} \cos \angle P_rP_oP_s,$$

so that

$$(3) \quad V_rV_s = \tfrac{1}{2}(a_{or}^2 + a_{os}^2 - a_{rs}^2).$$

On substituting (2), (3) into (1) and clearing of fractions in the determinant we get the desired formula

$$\text{Vol. } V_0V_1V_2V_3$$

$$= {}^1/_{24}\sqrt{2} \begin{vmatrix} 2a_{01}^2 & , a_{01}^2 + a_{02}^2 - a_{12}^2, & a_{01}^2 + a_{03}^2 - a_{13}^2 \\ a_{01}^2 + a_{02}^2 - a_{12}^2, & 2a_{02}^2 & , a_{02}^2 + a_{03}^2 - a_{23}^2 \\ a_{01}^2 + a_{03}^2 - a_{13}^2, & a_{02}^2 + a_{03}^2 - a_{23}^2, & 2a_{03}^2 \end{vmatrix}^{\frac{1}{2}}$$

PROBLEM DEPARTMENT

Edited by
T. A. BICKERSTAFF
University, Miss.

This department aims to provide problems of varying degrees of difficulty which will interest any one engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

Solutions

No. 8. Proposed by T. A. Bickerstaff:

There were 5 Sundays in February, 1920. Find the next four years in which February will have five Sundays.

Solved by Richard H. Stewart, Louisiana State University, Baton Rouge, La., and the proposer.

Obviously, the year must be a leap year. Therefore the length of the interval from February 1, 1920 to February 1 of any of these years must be a multiple of 1461 days. Also, the interval must have no fractions of weeks. Hence the interval must be a multiple of 7 days. Therefore the length of the first interval will be the least common multiple of 7 and 1461 days, or 10, 227 days or 28 years.

The years, consequently, are 1948, 1976, 2004, 2032, 2060 . . . .

Note: The continuity of this scheme will be broken where two consecutive leap years are more than 4 years apart. This will ccur first at the end of the 21st century, the years 2096 and 2104 being consecutive leap years.

No. 10. Proposed by the Problem Department Editor:

The digits of a three-digit number form an arithmetic progression. The digits of a number 10 less, form a harmonic progression. Find the number.

A. Solved by William E. Byrne, Virginia Military Institute, Lexington, Va.

Let the digits of the required number be

$$x-y,\ x,\ x+y$$

The digits of a number 10 less than the required number are

$$x-y,\ x-1,\ x+y.$$

By the conditions of the problem

$$\frac{1}{x-1}-\frac{1}{x-y}=\frac{1}{x+y}-\frac{1}{x-1} \quad \text{or} \quad x^2-y^2=x(x-1)$$

$$y^2 \qquad = x$$

We seek only solutions which are positive integers less than 10, and such that $(x-y)(x-1) \neq 0$ and $x+y<10$.

y	y^2	x	Remarks
1	1	1	Rejected since $(x-y)(x-1)=0$
2	4	4	which gives the number 246, a solution
3	9	9	Rejected along with all following values of y since $x+y>10$.

Conclusion There is one and only one solution, namely 246.

B. Solved by the proposer.

Let the digits be

a, a+d, a+2d

Then by the conditions of the problems

$$\frac{1}{a} - \frac{1}{a+d-1} = \frac{1}{a+d-1} - \frac{1}{a+2d}$$

or $d^2-d-a=0$

We seek only integral values of d numerically less than 10 and only positive integral values of a $\angle 10$.. such that $a+2d \angle 10$

Now since $d = \dfrac{1 \pm \sqrt{1+4a}}{2}$

We find, $a = 2$, or 6

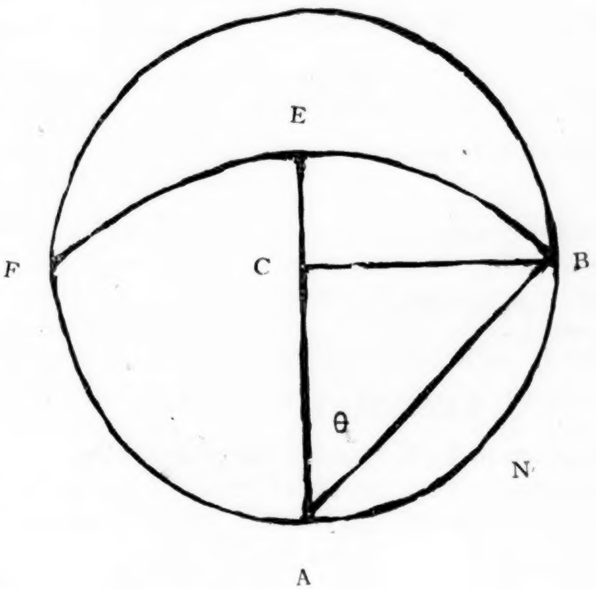
Whence $d = 2$, or -2

The number therefore is 246 or 642.

No. 13. Proposed by W. Van Parker, Mississippi Woman's College, Hattiesburg, Miss.

A goat is staked on the edge of a circular grass plot of radius R with a rope of length L. What must be the value of L in terms of R so that the goat can graze over half of the plot ?

Solved by J. T. Fairchild, Northern Ohio University, Ada, Ohio.



Let C represent the center of the grass plot and A the fixed end of the rope

Put AC = R, AE = L and let BAE = θ

Then AB = L = 2R cos θ

By the conditions of the problem, the area grazed upon is equal to

$$\frac{\Pi R^2}{2}.$$

$$\frac{\Pi R^2}{2} = \text{sector ABF} + 2 \text{ segment ANB}$$

$$= \text{sector ABF} + 2\,(\text{sector CAB} - \text{triangle CAB})$$

$$= L^2\theta + 2[\tfrac{1}{2}(\Pi - 2\theta)R^2 - \tfrac{1}{2}RL\sin\theta]$$

$$= 4R^2\theta\cos^2\theta + \Pi R^2 - 2R^2\theta - 2R^2\sin\theta\cos\theta$$

$$\frac{\Pi}{2} = 4\theta\cos^2\theta + \Pi - 2\theta - 2\sin\theta\cos\theta$$

$$= 2\theta(2\cos^2\theta - 1) + \Pi - 2\sin\theta\cos\theta$$

$$= 2\theta\cos 2\theta + \Pi - \sin 2\theta$$

$$\sin 2\theta - 2\theta\cos 2\theta = \frac{\Pi}{2}$$

Whence $\theta = 54°35'39''$

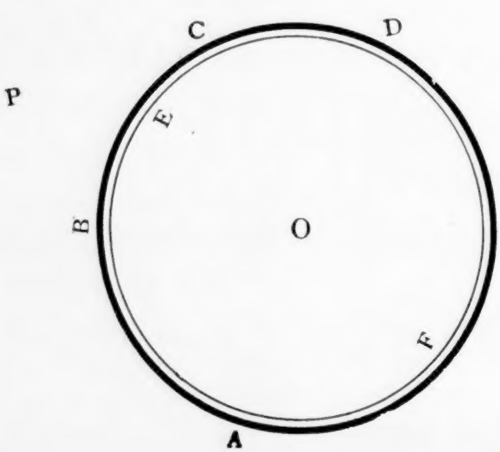
Therefore $L = 2R\cos 54°\,35'\,39''$

$= 1.158725R$

No. 11. Proposed by H. L. Quarles, University, Miss.:

Using compasses only, find the vertices of a square inscribed in a given circle. (From Newell and Harper: Plane Geometry, No. 16 on page 198).

Solved by the proposer:



Let the circle O be the given circle. Let A, B, C, and D be four of the vertices of a regular inscribed hexagon. With A and D as centers and AC as a radius, describe arcs intersecting in P. Using OP as a radius, mark off arcs AE, ED, and DF. A,E, D, and F are the required vertices.

It is easily seen that

$$OA = r$$

$$AP = AC = r\sqrt{3}$$

$$\therefore OP = r\sqrt{2}$$

No. 14. Proposed by Problem Department editor:

When I was born, my sister was one-fourth as old as my mother. She is now one-third as old as father. In four years, I shall be one-fourth as old as father. I am now one-fourth as old as mother. How old is each member of our family?

Solved by Velma R. Jenkins, State Teachers' College, Hattiesburg, Miss., and the proposer.

Let
W = my age
X = sister's age
Y = mother's age
Z = father's age

Then
$$\begin{cases} 4(4-W) = Y-W \\ 3X = Z \\ 4(W+4) = Z+4 \\ 4W = Y \end{cases}$$

Whence
W = 9.6 years
X = 16.8 years
Y = 38.4 years
Z = 50.4 years

Late Solutions

No. 9. Solved by William E. Byrne, Virginia Military Institutute, Lexington, Virginia.

Problems for Solution

No. 15. Proposed by William E. Byrne, Virginia Military Institute, Lexington, Virginia.

From Granville-Smith-Langley: Calculus page 113,

"What is the minimum value of

$$y = ae^{kx} + be^{-kx}?$$

Answer: $2\sqrt{ab}$

It happens that the answer is correct only under certain assumptions. What are these assumptions and what occurs in the other cases where $ab \neq 0$?

No. 16. Proposed by E. C. Kennedy, College of Mines and Mettalurgy, University of Texas, El Paso, Texas.

Find y^2 when

$$y = \sqrt{x - \sqrt{x - \sqrt{x - \sqrt{x - \sqrt{x - \ldots}}}}}$$ (Infinite number of radicals)

Shreveport, La., March 2, 1932.

Prof. S. T. Sanders,
Baton Rouge, La.

Dear Mr. Sanders:

I am sending you our Council program for our meeting in Oxford on the Eleventh and Twelfth. It follows:

"Some Problems In Plane Geometry"
Mr. Henry Schroeder, Ruston, La.

"The Gap Between High School and College Mathematics"
Miss Clyde Lindsey, Oxford, Miss.

"Some Mistakes I Made in Teaching High School Mathematics"
Mr. M. M. Abernathy, Shreveport, La.

"Educational Crises"
Dr. C. D. Smith, A. & M. College, Miss.

Roundtable discussions.

Truly,

J. T. HARWELL.